

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS General Certificate of Education Ordinary Level

ADDITIONAL MATHEMATICS

4037/12

Paper 1 May/June 2010

2 hours

Additional Materials: Answer Booklet/Paper

Graph paper (2 sheets)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.

Write your answers on the separate Answer Booklet/Paper provided.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 80.

UNIVERSITY of CAMBRIDGE
International Examinations

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} .$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n,$$

where *n* is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\csc^2 A = 1 + \cot^2 A.$$

Formulae for $\triangle ABC$

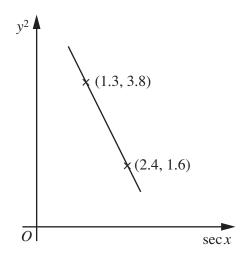
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$
.

$$\Delta = \frac{1}{2} bc \sin A.$$

- Find the coordinates of the points of intersection of the curve $y^2 + y = 10x 8x^2$ and the straight line y + 4x + 1 = 0.
- 2 The expression $6x^3 + ax^2 (a+1)x + b$ has a remainder of 15 when divided by x + 2 and a remainder of 24 when divided by x + 1. Show that a = 8 and find the value of b. [5]
- 3 Given that $\overrightarrow{OA} = \begin{pmatrix} -17 \\ 25 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$, find
 - (i) the unit vector parallel to \overrightarrow{AB} , [3]
 - (ii) the vector \overrightarrow{OC} , such that $\overrightarrow{AC} = 3\overrightarrow{AB}$. [2]

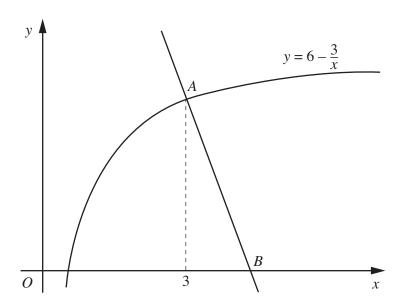
4



Variables x and y are such that, when y^2 is plotted against $\sec x$, a straight line graph passing through the points (2.4, 1.6) and (1.3, 3.8) is obtained.

- (i) Express y^2 in terms of $\sec x$. [3]
- (ii) Hence find the exact value of $\cos x$ when y = 2. [2]

5



The diagram shows part of the curve $y = 6 - \frac{3}{x}$ which passes through the point A where x = 3. The normal to the curve at the point A meets the x-axis at the point B. Find the coordinates of the point B. [5]

- **6** (a) (i) On the same diagram, sketch the curves $y = \cos x$ and $y = 1 + \cos 2x$ for $0 \le x \le 2\pi$. [3]
 - (ii) Hence state the number of solutions of the equation

$$\cos 2x - \cos x + 1 = 0 \text{ where } 0 \le x \le 2\pi.$$

[3]

- **(b)** The function f is given by $f(x) = 5\sin 3x$. Find
 - (i) the amplitude of f, [1]
 - (ii) the period of f. [1]
- 7 The table shows values of the variables p and v which are related by the equation $p = kv^n$, where k and n are constants.

ν	10	50	110	230
p	1412	151	53	19

(i) Using graph paper, plot $\lg p$ against $\lg v$ and draw a straight line graph.

Use your graph to estimate

(ii) the value of n, [2]

(iii) the value of p when v = 170. [2]

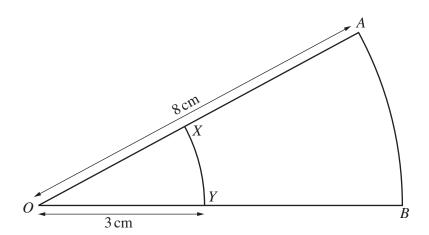
8 Given that $\mathbf{A} = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} -2 & 0 \\ 1 & 4 \end{pmatrix}$, find

(i)
$$3A - 2B$$
, [2]

(ii)
$$A^{-1}$$
, [2]

(iii) the matrix \mathbf{X} such that $\mathbf{X}\mathbf{B}^{-1} = \mathbf{A}$. [3]

9



The diagram shows a sector OXY of a circle centre O, radius $3 \,\mathrm{cm}$ and a sector OAB of a circle centre O, radius $8 \,\mathrm{cm}$. The point X lies on the line OA and the point Y lies on the line OB. The perimeter of the region XABYX is $15.5 \,\mathrm{cm}$. Find

(i) the angle AOB in radians, [3]

(ii) the ratio of the area of the sector OXY to the area of the region XABYX in the form p:q, where p and q are integers. [4]

10 A music student needs to select 7 pieces of music from 6 classical pieces and 4 modern pieces. Find the number of different selections that she can make if

(i) there are no restrictions, [1]

(ii) there are to be only 2 modern pieces included, [2]

(iii) there are to be more classical pieces than modern pieces. [4]

A particle moves in a straight line such that its displacement, x m, from a fixed point O on the line at time t seconds is given by $x = 12\{\ln(2t + 3)\}$. Find

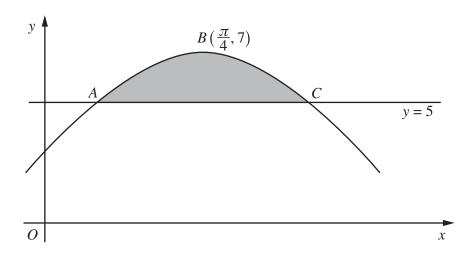
(i) the value of
$$t$$
 when the displacement of the particle from O is 48 m, [3]

(ii) the velocity of the particle when
$$t = 1$$
, [3]

(iii) the acceleration of the particle when
$$t=1$$
. [3]

12 Answer only one of the following two alternatives.

EITHER



The diagram shows part of a curve for which $\frac{dy}{dx} = 8\cos 2x$. The curve passes through the point $B\left(\frac{\pi}{4}, 7\right)$. The line y = 5 meets the curve at the points A and C.

- (i) Show that the curve has equation $y = 3 + 4 \sin 2x$. [3]
- (ii) Find the x-coordinate of the point A and of the point C. [4]
- (iii) Find the area of the shaded region. [5]

OR

A curve is such that $\frac{dy}{dx} = 6e^{3x} - 12$. The curve passes through the point (0, 1).

- (i) Find the equation of the curve. [4]
- (ii) Find the coordinates of the stationary point of the curve. [3]
- (iii) Determine the nature of the stationary point. [2]
- (iv) Find the coordinates of the point where the tangent to the curve at the point (0, 1) meets the x-axis. [3]

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